We model liquidity in housing markets. The model provides a simple characterization for the joint process of prices, sales, and inventory. We compare the implications of the model to certain properties of housing markets. The model can generate the large price changes and the positive correlation between prices and sales that we see in the data. Unlike the data, prices are negatively autocorrelated and high inventory predicts price appreciation. We investigate several amendments to the model. Informational frictions show promise.

**JEL codes:** D53, L11, R31

**Keywords:** house prices, liquidity, trading volume, trading frictions.

We study liquidity in housing markets. In housing markets, as in many markets with trading frictions, equilibrium prices and transactions volumes are closely linked. We construct a simple model of trade with matching frictions that provides a simple characterization of the joint process of prices, sales, and inventory. We use this model to analyze several puzzling features of housing markets.

Our baseline model is closely related to Shimer’s (2007) model of mismatch between workers and firms. Shimer was interested in the distribution of excess supply across markets. In addition, we are interested in the relationship between excess supply and equilibrium prices. In our model, there are many locations each with a fixed number of houses. In response to preference shocks, some homeowners become mismatched with their current locations. These homeowners put their current homes on the market and search for new homes elsewhere. Search is undirected. Each period potential buyers search randomly across markets. After buyers arrive, buyers and sellers bargain over prices. Prices depend on the local imbalance between buyers and sellers, as well as their outside options.

We would like to thank Humberto Ennis, Jonathan Heathcote, Haifang Huang, Ricardo Lagos, François Ortalo-Magné, Richard Peach, Robert Shimer, Gianluca Violante, and Pierre-Olivier Weill for helpful discussions and Peter Karadi for invaluable research assistance. This research was supported by the National Science Foundation under grant no. SES-0648545.

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Received December 10, 2008; and accepted in revised form March 5, 2010.

*Journal of Money, Credit and Banking*, Supplement to Vol. 43, No. 7 (October 2011) © 2011 The Ohio State University
Markets in this economy swing between periods of tight supply and excess supply. Sometimes there are more buyers than sellers, and the market is tight. Sometimes there are more sellers than buyers and there is excess supply. During periods of excess supply buyers extract surplus from sellers. Their ability to do so is limited by sellers’ option to wait and sell in the future. Sellers who make a sale must be at least as well off as sellers who do not. This links prices across periods. During periods of tight supply, sellers extract surplus from buyers. Their ability to do so is limited by buyers’ option to search elsewhere. This links prices across markets. In this sense, sellers—being tied to a location—are responsible for intertemporal arbitrage, whereas buyers—being free to search across markets—are responsible for intratemporal arbitrage.

The model generates a very simple characterization of the evolution of prices over time: whenever there is excess demand, sellers extract the maximal price; whenever there is excess supply, sellers must be indifferent between sales today and sales tomorrow. This simple characterization places strong restrictions on the dynamics of prices and allows for easy calibration.

We investigate the ability of this model to match four stylized facts regarding housing markets. The first is the positive autocorrelation in house price appreciation as documented by Case and Shiller (1989). This autocorrelation is puzzling because it does not appear to be explained by fundamentals.1 The second fact is that prices are volatile (Stein 1995, Ortalo-Magné and Rady 2006). The third is that price changes are correlated with the volume of transactions (Stein 1995, Ortalo-Magné and Rady 2006). The final fact is the negative correlation between the inventory of unsold homes and future price appreciation. Several authors include lagged measures of the inventory of unsold homes in aggregate pricing equations (e.g., Peach 1983) and find that the effect was negative. As this fact seems to be less appreciated in the literature, we devote a little time to extending it. We consider a sample of U.S. cities, and ask whether the proportion of vacant homes for sale helps to forecast subsequent price changes in the cross-section. We find that higher vacancies predict lower rates of price appreciation.

Our baseline model has mixed success. On the positive side, it generates a positive correlation between price changes and the volume of transactions. A large number of transactions leads to low inventory and high current prices. Calibrated versions of the model can also deliver sizable movements in prices as shifts in market tightness lead to shifts in surplus between buyers and sellers.

On the negative side, our baseline model generates negative autocorrelation in price changes and a positive correlation between the inventory of unsold homes and future price appreciation. The reason for this failure is instructive and likely to pose problems for any model that jointly models prices and inventories. When the inventory of unsold homes is zero prices are at their maximum and can only fall. Positive shocks therefore

1. Case and Shiller (1989) argue that serial correlation in rents does not explain inertia in price changes. In order to control for difficulties in measuring rents, Glaeser and Gyourko (2006) use wages to proxy for fundamentals. They also are unable to generate significant positive serial correlation in price changes.
tend to raise prices today and lead to falling prices in the future. Conversely, when inventories of unsold homes are high, prices will tend to be low. The expected rate of price appreciation, however, must be high in order to compensate sellers who must wait to sell in the next period. A shock that raises inventory causes price to fall today and rise tomorrow, thereby leading to negative short-run serial correlation in price changes and a positive correlation between inventories and future price appreciation.

One possible explanation for these facts is that prices respond sluggishly to market conditions. Sluggish adjustment leads naturally to autocorrelation and sluggish prices may lag reductions in inventory. To generate sluggish price adjustment, we incorporate an informational friction into our model. In particular, we assume that prices are posted before demand is realized, as in the sequential service economies of Prescott (1975) and Bental and Eden (1993). Demand uncertainty leads to price dispersion in each market. Some sellers will charge low prices and sell with high probability, whereas other sellers will charge high prices and sell only if demand is sufficiently high. Prices in this version of the model do not incorporate all contemporaneous information, causing price changes to lag inventory dynamics. When demand is unexpectedly high, prices will rise, but this rise will be tempered by the fact that some homes sold at low prices before high demand was realized. During the next period when inventory is known to be low, prices may rise further.

This version of the model performs much better. A calibrated version of our sequential service economy generates significant positive autocorrelation in price changes and a correlation between inventories and subsequent price changes that is close to zero. A significant gap, however, remains between theory and data. First, the effects of sluggishness are short-lived. While the correlation between price changes in consecutive quarters is positive, the autocorrelation function quickly turns negative. Second, informational frictions make the price process much more sluggish, which reduces the volatility of house prices.

While our focus is on the time series properties of prices, the models that we consider shed light on other features of the trading process. The sequential service economy captures the ambiguity in the relationship between price and time-on-the-market. In any given market in any given period, high prices are correlated with a longer time on the market, since high priced homes are less likely to sell. Across markets and time, however, high prices are correlated with low inventory and hence a greater probability of sale. The sequential service economy also makes the prediction that price dispersion is correlated with sales. We know of no empirical evidence on this issue.

The model is complementary to search models of housing such as Wheaton (1990). These models also predict, a positive relationship between price and market tightness. They also generally imply that tight markets face expected future price declines. There are several differences between the two frameworks. First, in search models buyers and sellers match bilaterally. Brokers and multiple listing services, however, give agents the ability to sample more widely. In our framework, buyers and sellers see all of the houses that are for sale in their market at any given point in time. Second, search models tend to keep bargaining power fixed so that all price fluctuations are
generated by fluctuations in the outside option. In our framework, bargaining power shifts between buyers and sellers as market conditions change. Third, search models cannot easily accommodate alternative market structures such as in the sequential service economy. Interestingly, search models do not seem to generate the ambiguity in the relationship between price and time on the market.\(^2\)

Section 1 presents and analyzes the basic model. Section 2 discusses some of the comparative static properties of the model. We calibrate the model in Section 3. Section 4 presents some evidence from U.S. cities on the correlation between market tightness and subsequent price appreciation. Section 5 considers a version of the model with prices that are posted before demand is realized. Section 6 concludes.

### 1. A MODEL OF A HOUSING MARKET

We present our baseline model later. In subsequent sections we show how varying the timing of events alters the properties of the equilibrium.

Our focus is on the implications of trading frictions for price behavior. We keep other aspects of the model as simple as possible. As in the Lucas (1978) asset pricing model, we consider an economy in which equilibrium quantities are clear. What is not clear is how to price the asset, housing. To focus on fluctuations in price that are driven by shifts in bargaining power between buyers and sellers, we remove all other sources of price fluctuations. There are only two sources of heterogeneity: the random shocks to homeowner preferences and the random search process. Otherwise all houses and all agents are identical. Finally, we also remove all life-cycle and financial considerations. We assume that agents always have enough assets or borrowing ability to purchase a house that they desire.

#### 1.1 Environment

Time is discrete and indexed by \( t \). The economy is composed of a continuum of housing markets of measure one. Markets are indexed by \( i \in [0, 1] \). Each market holds a finite number of homes. We denote this number by \( H \). \( H \) is an integer. There is a population of \( \bar{N} \) agents per market. \( \bar{N} \) is a real number.

Agents may own multiple homes, but each home has a single owner. The owner of a home may either be well matched or mismatched with the home. An owner who is well matched to a home derives \( \gamma \) units of utility from the home in each period. An owner who is mismatched derives zero utility. Matches evolve over time. New owners are always well matched in the period in which they purchase their homes. At the very beginning of each period some well-matched owners become mismatched. The probability that an owner who is well matched with a home becomes mismatched

\[ \text{Probability} = \frac{1}{10} \]

2. In Wheaton’s (1990) model tight markets lead to high prices and quick sales. In Albrecht et al. (2007) sellers who remain on the market become desperate and reduce prices; again, price and time-to-sell are negatively related. See also Krainer (2001) and Novy-Marx (2009).
at the beginning of period is constant over time and equal to $z$. The dissolution of matches is independent across owners and time. For simplicity, mismatched owners remain mismatched forever.

We make several assumptions on the structure of matches so an owner chooses to search if and only if the owner is not well matched to a home. First, we assume that an agent can derive utility from only one home at a time and that an agent cannot search proactively and stockpile well-matched homes in anticipation of the agent’s current match dissolving.\(^3\) Well-matched owners therefore have no incentive to search for housing. Second, we assume that mismatched owners are also mismatched with other homes in their market, so that they gain nothing by swapping homes among themselves.\(^4\) They must search elsewhere. Third, we allow mismatched agents to search for new homes while they sell their old homes. In fact, we do not link in any way an agent’s decision to sell their old home and purchase a new one.\(^5\) The result of these assumptions is that agents attempt to sell all of their mismatched houses, and if they are not well matched, they search until they purchase a single new home.

Search is a black box. We model the outcome of the search process, rather than search itself. We assume that each searcher can sample only one market per period. Let $N$ denote the average number of searchers per market. Note $N < \bar{N}$ since only mismatched owners search. The outcome of the search process is a probability density $\phi(n; N)$ that gives the probability that $n$ buyers arrive in a market conditional on there being a mass $N$ of searchers in the economy. Note that $n$ is an integer, since the number of homes in a market $H$ is an integer. We will consider several special cases of $\phi$.

Shimer makes the assumption of “random search”:

$$\phi(n; N) = e^{-N} \frac{N^n}{n!}.$$  

This is the density that arises when a mass $N$ agents each choose uniformly over a unit mass of markets. Another convenient functional form is “exponential search”:

$$\phi(n; N) = (1 - \pi_N)\pi_N^{n-1}, \quad \pi_N \in (0, 1).$$

\(^3\) An assumption that would justify ruling out proactive search is that agents do not know where to search until their current match dissolves. Suppose that locations are grouped into different types and that only at the time a match dissolves does an agent learn his new type. By increasing the number of types, we can make it prohibitively expensive to buy the insurance that proactive search provides.

\(^4\) Hanushek and Quigley (1979) argue that separations are mainly due to exogenous life events—loss of job, change in family composition. Many people, however, move within a metropolitan area. Here one might think of the market more narrowly as both a location and a particular type of housing (efficiency, one-bedroom, two-bedroom, yard/no yard).

\(^5\) Linking the purchase and sales decisions can lead to anomalous results. For example, if agents must sell their current home before purchasing a new one, then house prices may be negative for reasonable parameterizations. Agents are willing to pay buyers to take their homes so that they may enter the search process and purchase another home at a negative price. Decoupling the sales decision links house prices more closely to the value of fundamentals, in this case the present value of $\gamma$. 
Here $\pi_N$ is set so that the average number of buyers $(1 - \pi_N)^{-1}$ is equal to $N$. This formulation will prove convenient when we consider uncertain demand. In this formulation the arrival of $n$ buyers says nothing about the arrival of the $(n + 1)$st buyer. Note that search is not directed in any of these formulations. The simplicity of the model is due to the fact that prices do not affect search. This allows us to solve for allocations independent from prices.

Prices are determined by Bertrand competition among sellers. The total number of buyers and sellers in market $i$ at date $t$ is known when sellers set prices. After prices are set, buyers queue up randomly and choose sequentially whether to purchase a home or to wait and search again in the next period. If a buyer chooses to purchase a home, the buyer becomes the new well-matched owner. Unsold homes remain on the market in the next period. We assume that owners of unsold homes incur a carrying cost $l$ units of utility (which may be equal to zero) each period that their homes remain unsold. This carrying cost is in addition to the delay in receiving payment for the home.

The timing of events within a period is as follows: owners become mismatched, buyers arrive, sellers set prices, and trade takes place. Buyers and sellers make purchase and pricing decisions in order to maximize the present value of utility. Prices are quoted in units of utility. Utility in period $t$ is the sum of utility from housing plus the price of any homes sold less the price of any home purchased. Future utility is discounted by a factor $\beta \in (0, 1)$.

### 1.2 Solution

The model is set up so that we can solve for quantities first and then prices. Note that it will be efficient for all potential trades to take place each period. Trade simply replaces a mismatched owner with a well-matched owner.

We look for a steady state with a stationary cross-sectional distribution of prices and vacancies. In this steady state, the value of search will be constant, as it is in the search model of Lucas and Prescott (1974). We restrict our attention to equilibria that are symmetric across locations and in which sellers follow pure pricing strategies that are functions only of the current state of their market. This allows us to restrict our attention to a single location when solving for equilibrium prices.

In what follows, we first solve for the evolution of sales and vacancies in each market as a function of the separation rate $z$ and the arrival rate $\phi$. We then characterize the cross-sectional density of vacancies. Finally, we solve the Bertrand pricing game among sellers given the evolution of sales and inventory and confirm that efficient

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6. An alternative formulation would be to have sellers set prices before demand is realized. We will return to this issue later.
7. The linearity of utility in prices means that we do not need to track the distribution of wealth.
8. In some search models, the social planner wishes to prevent some trades from taking place. This alters the distributions against which buyers search and may improve future matches. In the present model, all matches have the same value. Prices are merely transfers between buyers and sellers. There is no social benefit to increasing the inventory of unsold homes.
trade is in fact an equilibrium. We use these pricing strategies to solve for the value of owning a home and the value of search.

**Quantities.** We drop the subscripts $i$ and $t$ except where they are necessary to avoid confusion.

It is useful to define two sets of sellers. Let $b \in \{0, 1, \ldots, H\}$ denote the number of sellers at the beginning of the period, after the match shock, but before trade. Let $e \in \{0, 1, \ldots, H\}$ denote the number of sellers at the end of the period after trade has taken place.

We create two transition matrices. The first matrix, $P_z$, describes transitions from $e_{t-1}$ to $b_t$. Since $e_{t-1}$ is the number of unsold homes at the end of period $t-1$ and $b_t$ is the number of homes for sale during period $t$, the difference, $b_t - e_{t-1}$, reflects the arrival of new sellers. Given $e_{t-1}$ there are $H - e_{t-1}$ well-matched owners in the market. As matches dissolve with probability $z$ and the dissolution of matches is independent across owners, the distribution of new sellers is a binomial distribution with parameters $z$ and $H - e_{t-1}$. $P_z$ is an $H+1 \times H+1$ matrix, indexed so that the $(i, j)$th element gives the probability of moving from $e = i - 1$ to $b = j - 1$. It is upper triangular since there is always a weakly positive flow of new sellers, $b_t \geq e_{t-1}$.

The second matrix describes transitions from $b_t$ to $e_t$. Given the assumption of efficient trade, the short side of the market determines the number of trades. The number of unsold homes at the end of period $t$ is equal to $b_t - n_t$ or 0, whichever is greater. This gives rise to a transition matrix $P_{n,N}$ which depends on $\phi(n, N)$. $P_{n,N}$ is an $H+1 \times H+1$ matrix. The $(i, j)$th element gives the probability of moving from $b = i - 1$ to $e = j - 1$. $P_{n,N}$ is a lower triangular matrix, since the number of buyers is weakly positive. For $i \geq j > 1$, $P_{n,N}(i, j) = \phi(i - j, N)$, the probability that $n = i - j$. For $j = 0$, $P_{n,N}(i, j) = \sum_{k>0} \phi(k, N)$, which is the probability that $n \geq b$.

From $P_{n,N}$ and $P_z$, we can derive the transition matrices form $e_{t-1}$ to $e_t$ and from $b_{t-1}$ to $b_t$. Transitions from $e_{t-1}$ to $e_t$ are governed by $P_e = P_z P_{n,N}$. Transitions form $b_{t-1}$ to $b_t$ by $P_b = P_{n,N} P_z$. Note that since $P_z$ has strictly positive elements along and above the diagonal and $P_{n,N}$ has strictly positive elements along and below the diagonal, $P_b$ and $P_e$ are everywhere strictly positive.

Let $\lambda^e$ be a $(H+1) \times 1$ vector that gives the steady-state, cross-sectional density of end-of-period sellers. The $i$th row gives the fraction of markets with $e = i - 1$ sellers (the first row is associated with $e = 0$). We have

$$\lambda^e P_e = \lambda^e$$

or

$$(I - P_e^\prime)\lambda^e = 0.$$
$\lambda^e$ is therefore the eigenvector associated with the unit eigenvalue of $P_e$. Since all of the elements of $P_e$ are strictly positive, there exists a unique $\lambda^e$ for any $N$ (Sargent and Ljungqvist, 2004, Theorem 1, p. 33). Similarly we may define $\lambda^b$ as a density over $b$. We have

$$\lambda^b' P_b = \lambda^b$$

and

$$\lambda^e' P_e = \lambda^b$$

$$\lambda^b' P_{n,N} = \lambda^e.$$

It remains to determine $N$. To do this, we calculate the population in two different ways. The steady state value of $N$ equates these:

$$\tilde{N} = H - Z \cdot \lambda^b + N. \quad (1)$$

The left-hand side restates the assumptions that there are $\tilde{N}$ agents per market and a measure one of markets, so that the total population is $\tilde{N}$. The right-hand side is composed of three terms. $H$ is the number of homeowners. $Z$ be a row vector whose elements are $\{0, 1, \ldots, H\}$, so that $Z \cdot \lambda^b$ is the number of mismatched homes. $H - Z \cdot \lambda^b$ is therefore the number of well-matched homeowners. Owners that are not well matched are searching. There are $N$ of these. Note that $Z \cdot \lambda^b$ is decreasing in $N$, since increasing $N$ leads to more sales and hence fewer unmatched homes. The right-hand side is therefore increasing in $N$. There is a unique $N$ that satisfies (1).10

This completes the analysis of quantities. We now turn to prices.

**Prices.** Price setters play a Bertrand game. We look for an equilibrium in pure strategies. Since all sellers are alike, each sets the same price. The state variable that matters for pricing is $e$, the number of unsold homes at the end of the period. $e$ determines the reservation value for both buyers and sellers. Sellers who wait to sell in the future only care about the number of unsold homes that remain at the end of the current period. Buyers care only about the price that they may receive if they sell their home in the future. This expectation depends only on $e$.

We define a number of useful expressions. Let $v$ denote the present value of being a buyer searching this period. $v$ is constant with a stationary cross-sectional distribution of prices and vacancies. Let $V$ be an $(H + 1) \times 1$ vector that gives the value of owning a home when there are $e$ unsold homes at the end of the period. The $i$th row gives

10. Given $N = 0$, there are no sales so all homes are vacant in the stationary distribution. The right-hand side of (1) is zero. At $N = \tilde{N}$, $Z \cdot \lambda^b < H$, so that the right-hand side of (1) is greater than $\tilde{N}$.

11. Suppose that we start out of steady state, how would the economy behave? Would it converge? Would it converge monotonically? At what rate? The analysis of convergence is made difficult by the fact that $N$ and hence $P_{n,N}$ may change over time. This is a time inhomogeneous Markov chain and standard convergence theorems do not apply.
the value of owning a home when \( e = i - 1 \). Let \( p \) be an \((H + 1) \times 1\) vector of equilibrium prices. Again the \( i \)th row gives the price of a home conditional \( e = i - 1 \) unsold homes at the end of the period.

There are two cases to consider: \( e = 0 \) and \( e > 0 \).12 If \( e = 0 \), then \( n \geq b \). There are (weakly) more buyers than sellers, and every seller makes a sale. Sellers in this case behave collectively as monopolists. They extract the entire surplus from buyers. Price is equal to a buyer’s maximum willingness to pay. As a buyer receives \( \beta v \) from search in the next period, we have:

\[
V_0 - p_0 = \beta v,
\]

where \( V_0 \) is the value of owning a home when \( e = 0 \), and \( p_0 \) is the price in this case. The left-hand side is the value to a buyer of purchasing a house today. The right-hand side is the value of search tomorrow. \( p_0 \) is set to equate these two options.

If \( e > 0 \), there are more sellers than buyers. Some homes go unsold. With Bertrand competition among identical sellers, sellers must be indifferent between selling today and holding on to their homes in order to sell in the future. It is shown that in the Appendix that this implies:

\[
p_e = -l + \beta x'_e p_e, \quad e > 0,
\]

where \( x_e \) is a column vector of zeros with a one in the \((e + 1)\)st row. The left-hand side is the return from selling a house today. The right-hand side is the value of waiting. The value of waiting is equal to the expected present value of the sales price tomorrow, \( \beta x'_e p_e p \), less the carrying cost \( l \). The intuition here is the same as in the standard McCall search model: tomorrow the seller will either be indifferent between selling and waiting (if \( e > 0 \)) or strictly prefer selling (if \( e = 0 \)); hence, we can replace the continuation value with the value of selling.

The next two subsections characterize \( v \) and \( V \).

**The value of owning a home.** We now consider the value of owning a home when the owner is well matched. Let \( P^+ \) be an \((H + 1) \times (H + 1)\) matrix that gives the probability of transitioning from \( e_{t-1} \) to \( b_t \), conditioning on the fact that at least one owner becomes mismatched, and let \( P^- \) be an \((H + 1) \times (H + 1)\) matrix that gives the probability of transitioning from \( e_{t-1} \) to \( b_t \), conditioning on the fact that at least one owner remains well matched.

The value of owning a home is equal to\(^{13}\)

\[
V = \gamma + \beta \left[ (1 - z) P^-_{N,N} V + z P^+_N N (p + v) \right].
\]

---

12. This is similar to Shimer (2007). Shimer has two wages depending on whether or not there is excess supply of workers. Here we have several prices. The difference is that Shimer only has spot markets whereas housing is a durable that may be sold today or sold tomorrow and when sold may be resold at some future date.

13. Note that \( V_H \) is not well defined in this expression, since a person cannot be well matched in a market in which all owners are mismatched. It may be set arbitrarily.
The value of owning a home is equal to the flow value $\gamma$ plus the present value of an optional policy in the future. In the future, with probability $1 - z$, the owner will remain well matched. $P^z P_{n,N} V$ is the expected value of owning a home conditional on remaining well matched. With probability $z$, the owner will become mismatched. $P^z P_{n,N} p$ is the expected value of an optimal sales policy, $P^z P_{n,N} v$ is the expected value of search. Both expectations are conditional on becoming mismatched.

The value of search. We close the model with an expression for $v$. A buyer assigned to market $i$ receives a payoff that depends on the excess supply in that market. If $n < b$ then the buyer makes a purchase and receives a benefit $V_{b-n} - p_{b-n}$. If $b > n$, the buyer receives $\beta v$ regardless of whether or not the buyer makes a purchase since the buyer is indifferent between a purchase and search. To solve for $v$, we integrate over the densities of $b$ and $n$:

$$v = \frac{1}{N} \sum_b \lambda^b \left[ \sum_{n < b} \phi(n; N) n (V_{b-n} - p_{b-n}) + \sum_{n \geq b} \phi(n; N) n \beta v \right].$$

The probability that the market has $b$ sellers is $\lambda^b$. The probability of arriving in a market with $n$ buyers is $n \phi(n; N)/N$. These two probabilities are independent, we therefore first sum over $n$ and then over $b$.

This completes the solution to the model.

2. DISCUSSION

Several properties of this solution are worth noting. First, we can iterate (3) forward. Sellers are indifferent between selling waiting in every period that $e > 0$. It follows that sellers are indifferent between selling today and waiting until $e = 0$.

Let $T$ be a stopping time defined as the first time that $e = 0$. We have

$$p_e = E \left\{ \beta^T p_0 - \frac{(1 - \beta^T)}{1 - \beta} \left| e \right. \right\}. \quad (4)$$

This characterization immediately implies that $p$ is monotonically decreasing in $e$; as $e$ rises so does $T$.

An immediate consequence of equation (4) is that price is negatively correlated with end of period inventory. Price fluctuations result in shifts in surplus between buyers and sellers. In this model the fundamental value of the house is a constant flow $\gamma$. Given that the probability of becoming mismatched is likely to be quite low (below we calibrate it to be 2% per quarter), buyers valuations $V_e$ should not change much as $e$ changes. Sellers therefore get a better deal in tight markets and buyers get a better deal in loose markets.

Second, inventory dynamics lead naturally to mean reversion in the long run. Given that $P_e$ has no nonzero elements, it follows immediately that the expected
inventory level converges to the average inventory level as determined by the probability density $\lambda$. Higher inventory levels are expected to fall. Lower inventory levels are expected to rise. This leads to negative autocorrelation in price changes. If inventories fall today, prices rise, leading to expectations of future increases in inventories and reductions in prices.

An immediate consequence is that price changes are predictable. In fact there are two cases, depending on whether or not there are end-of-period inventories. When end-of-period inventories are positive, equation (4) implies that the expected gross rate of price appreciation is:

$$E_t(p_{t+1}/p_t) = \frac{1}{\beta} + \frac{l}{\beta}p_t.$$

When $l$ is equal to zero price appreciation is equal to $1/\beta$, the risk-free rate. This is reminiscent of the Hotelling (1931) model of natural resource extraction. As in the Hotelling model, if prices were to rise faster, each seller would have an incentive to wait. This would push prices up and reduce the rate of price appreciation. If prices were to rise at a slower rate, each seller would have an incentive to sell today. Competition among sellers would bid prices down and increase the rate of appreciation.

When end-of-period inventories are zero, $p_0 = V_0 - \beta v$ and the expected price is $x^\prime_0 P e p$. Recall $x_0$ is a column vector of zeros with a one in the first row. Since $p_e < p_0$ for all $e > 0$, prices are expected to fall. Unlike when $e > 0$, all sellers are already making sales so there is no additional supply of mismatched owners to push down prices. One might ask whether matched owners have an incentive to enter the market and take advantage of the high prices. Matched owners, however, are identical to the buyers. Since the buyers are willing to buy at price $p_0$ in spite of the prospect of falling prices, matched owners are also willing to remain in their homes. Although prices are predictable, the search friction prevents arbitrage.

Third, while expected price changes are equal to $1/\beta$ when $l = 0$, actual price changes are likely to be much greater. Consider a market with one unsold home. The upside is bounded by $p_0$. The downside is effectively unbounded. The price must therefore be less than $\beta p_0$ in order to generate an expected increase of $1/\beta$. The model therefore has the potential to generate large price changes.

Finally, it follows from equations (2) and (3) that equilibrium considerations affect prices only through $p_0$. Hence, many changes in the structure of the model will not affect the properties of prices. Note that when $l = 0$, the characterization of the price process becomes particularly simple. Let $\rho = pl/p_0$. Now $\rho_0 = 1$, and when $l = 0$, equation (3) implies that

$$\rho_e = \beta x^\prime_e P e \rho$$

for $e > 0$. $\rho$ depends only on $\beta$, $z$ and $\phi(n; N)$ and is independent of $V$ and $v$. 
3. CALIBRATION

To calibrate the model, we first need to determine the time period. The natural time period is the one over which prices are fixed. Unfortunately, there is not much evidence on the time between price adjustments. Knight (2002) reports that an average time of 3.5 months before a list price change for a sample of homes in Stockton, California during 1998. Merlo and Ortalo-Magné (2004) report an average of 11 weeks until the first price change in their sample of British properties. We therefore take the time period to be a quarter.

We need to calibrate $N$, $z$, $\beta$, $H$, $\gamma$, and $l$. We set $\beta$ to 0.99 implying a discount rate of about 4% per year. We set $z$ to match data on turnover. Dieleman et al. (2000) find that the turnover rate is about 8% per year for urban home owners. We therefore set $z$ equal to 0.02 for our quarterly model. We choose $N$ and $H$ to match data on the time to buy and the time to sell. Anglin and Arnott (1999) report that time to buy and time to sell are both in the neighborhood of a quarter.\textsuperscript{14,15} This results in $H = 67$ and $N = 2.29$. The small value of $H$ is not troubling since there is nothing in the model that says the market is large. We can think of markets in the model as being quite specific: two bedroom homes located in a certain neighborhood. We normalize $\gamma$ to one and set $l$ to zero.

Figure 1 presents price as a function of end-of-period vacancies. As expected, price declines monotonically with vacancies. Figure 2 presents $\lambda^*$, the steady-state density of end-of-period vacancies across markets. Most markets have no vacancies. This is a direct consequence of the calibration of the time to sell. Given that time to sell is equal to one period, there must be lots of markets that stock out each period.

On the positive side, the model can generate reasonably large price changes. The mean absolute percentage change in prices in the calibrated model is 2.8% per quarter.\textsuperscript{16} The median is zero, reflecting the large number of markets that stock out in consecutive periods. The standard deviation is about 1.5%. The model also generates a positive correlation between sales and the change in house prices. The correlation is about 0.3.

The model misses badly on two other facts. The autocorrelation in price changes is $-0.23$. The correlation between end-of-period vacancies and future price changes is just over 0.3. Each is the opposite of what is commonly found in the literature. These correlations reflect the mean reversion in inventories discussed above.

\textsuperscript{14} Given that we have chosen the unit of time to be a quarter, we would need all homes to sell every period to match a time to sell of one quarter. We assume that sales are spread evenly throughout the period, so that immediate sales could as 0.5 quarters and sales after $n$ periods count as $n + 0.5$ quarters.

\textsuperscript{15} Time to sell in the data conditions on a sale taking place. It ignores withdrawals. There are no withdrawals in the model because mismatch is an absorbing state. Hendel et al. (2009) consider these issues.

\textsuperscript{16} Model moments were calculated off of a 10,000 period simulation of the model.
4. SOME EVIDENCE ON MARKET TIGHTNESS AND PRICE APPRECIATION

A strong prediction of the baseline model is that inventories are positively correlated with future price appreciation. High inventories are associated with low current prices and high future appreciation. Peach (1983) finds the opposite using aggregate data. Given this divergence between model and data, we update his evidence.

We consider an unbalanced panel of U.S. cities. The time period is 1988 to 2005. The data are annual since our measure of inventory is annual. We measure market tightness as the proportion of homes that are vacant and for sale. These data are from the Bureau of Economic Analysis. This measure obviously ignores homes that are occupied and for sale. Our maintained assumption is that these are highly correlated with vacancies. We take the S&P Case-Shiller house price index as our measure of house prices. This is a repeat sales index. There has been much discussion of the potential biases in this index. Unfortunately, there is no other widely available index that is obviously better. The data begin in 1987 with 14 cities. Coverage increases to 20 cities over the sample period.
We estimate the following relationship

$$\Delta p_{it} = \alpha_i + \beta t + \gamma \Delta p_{i,t-1} + \delta v_{t-1} + \varepsilon_{it}. \tag{5}$$

Here $\Delta p_{it}$ is the change in the log of the price index in city $i$ between dates $t - 1$ and $t$. $\alpha$ is a city fixed effect meant to capture differences in the natural vacancy rate. $\beta$ is a time effect meant to capture common influences such as the state of the national business cycle or the level of interest rates. $v$ is vacancies. We lag vacancies since we are interested in the predictive power of vacancies for future price changes. $\varepsilon$ is the error term.

Table 1 presents the results from estimating equation (5). We see positive serial correlation in price changes. We also see that vacancies have a significantly negative correlation with subsequent price changes.

The results are robust to changes in specification. Dropping the last year (2005) increases the coefficient on $v_{t-1}$ to $-1.96$ with a standard error of 0.42, indicating that the effect is stronger before the recent run up in prices. Lagging vacancies by two periods also increases the coefficient on vacancies; the coefficient rises to $-1.6$ with a standard error of 0.6. Adding an additional lag price change also does not affect the
TABLE 1

PANEL ESTIMATES OF PRICE APPRECIATION ON LAGGED APPRECIATION AND LAGGED VACANCIES

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p_{t-1}$</td>
<td>0.833</td>
</tr>
<tr>
<td>$v_{t-1}$</td>
<td>-1.265</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the change in the log of the price index. The data period is 1988–2005. The regression includes city effects and time effects.

results. Prices appear to decline following high vacancies, the opposite of what the model predicts.\(^{17}\)

5. POSTING PRICES BEFORE DEMAND IS REALIZED

One possible reason for the positive autocorrelation in price changes and for the negative correlation between inventories and future price changes in the data are that prices respond sluggishly to market conditions. Gradual adjustment of prices tends to lead to positive autocorrelation in price changes. Gradual adjustment of prices may cause price changes to lag inventory changes.

To generate sluggish price adjustment, we incorporate an informational friction into our model. We alter the above analysis by assuming that sellers post prices before demand is realized. Sales remain efficient, so that the distributional dynamics are the same as in the baseline model. See the “Quantities” subsection. Prices, however, do not completely incorporate market conditions.

When sellers post prices before demand is realized, it will not be optimal for any two sellers to charge the same price. If they did, then one of the sellers could lower his price slightly and by taking only a small loss in revenue greatly increase his probability of making a sale. Demand uncertainty therefore leads to a distribution of distinct prices as in the analysis of Prescott (1975). Since sellers are identical, each seller will be indifferent between the entire set of prices.\(^{18}\) What sellers with low prices lose in terms of revenue, they make up in the probability that they make a sale.

Consider now a market with $b$ sellers. We solve for the optimal pricing policy by induction. Consider first the seller who charges the highest price. This seller makes

17. In his discussion of this paper, Francois Ortalo-Magné presented evidence from a single neighborhood in Madison, Wisconsin that appeared to support the implications of the baseline model. In his data, a high inventory of unsold homes appeared to be correlated with low prices and future price appreciation. This raises the possibility that the relationship between prices and vacancies differs at different levels of aggregation. The baseline model’s predictions may be more applicable to very local markets.

18. This pure strategy Nash equilibrium imposes a minimum amount of coordination among sellers. They must sort themselves among the various prices offered during the period. The analysis would be different if sellers did not coordinate but rather randomized independently across prices. In this case, it would be possible for inefficiencies to arise. If, for example, all sellers charged the monopoly price and there were only a few buyers, then the buyers might prefer not the make a purchase and to search again in the next period. This would break the independence of quantities from prices.
a sale if and only if there are at least as many buyers as sellers. The seller will set the monopoly price:

\[ p_0 = V_0 - \beta v. \quad (6) \]

Note we have indexed price \( p \) to the number of sellers left after the current sale is made and \( V \) to the number of homes left on the market. This will facilitate comparison with the last section.

We now turn to the induction step. Consider the seller with the \( j \)th highest price where \( j \geq 2 \). This seller must be indifferent between charging \( p_{j-1} \) and making a sale if there are at least \( b + 1 - j \) buyers and charging the next highest price \( p_{j-2} \) and making a sale if there are at least \( b + 2 - j \) buyers. The indifference condition turns out to be simpler if we work with a search process that is governed by an exponential density

\[ \phi(n) = (1 - \pi_N)\pi_N^{n-1}, \quad \pi_N \in (0, 1). \]

We show in the Appendix that this leads to the following indifference relation for the seller with the \( j \)th highest price

\[ p_{j-1} = \pi p_{j-2} + (1 - \pi) \left[ -l + \beta x_{j-1} \right]. \quad (7) \]

With the exponential distribution, the probability of an additional buyer is independent of how many buyers have already shown up in the market. This allows us to condition on there already being \( b + 1 - j \) buyers. In this event, the seller with the \( j \)th highest price weighs making a sale at price \( p_{j-1} \) with probability one against the possibility of trading places with the next highest priced seller and making a sale at price \( p_{j-2} \) in the event that there is at least one additional buyer. The additional buyer arrives with probability \( \pi \). With probability \( 1 - \pi \), the higher price seller will wait to make a sale in the next period. This involves the flow cost \( l \) and the expected benefit \( x_{j-1} \), which is the expected value of making the first sale in the next period.

With the exponential density, at least one buyer is guaranteed to show up. Equations (6) and (7) determine prices as functions of \( V^e \) and \( v \). We can solve for \( V^e \) and \( v \) in a manner similar to Section 2.

The following proposition illustrates the sense in which the lowest price when demand is uncertain is related to the expected price when demand is known. The proof is contained in the Appendix.

**Proposition** Consider two economies with identical values of \( \pi \) and \( V_0 - v \) but differing in when demand is realized. Let \( p \) denote the equilibrium price in the economy with uncertain demand and \( \hat{p} \) denote equilibrium price when demand is known before setting the price. Then,

\[ p = P_n \hat{p}. \]
While the prices in the two markets are related, the stochastic properties of the price process are not the same. Transactions do not fully reflect current demand when prices are set before demand is realized. Suppose that beginning-of-period inventory is high. Then some sales will be made at low prices even if demand turns out to be high. The full effects of high demand will only be incorporated in a few higher priced sales. The average price during the period will be an average of both the high and low prices. This averaging mutes the price increase during high demand periods. Since demand was high, end-of-period inventories will be low, and next-period prices will tend to be high on average. In this way, the model can possibly generate positive serial correlation in price changes and a negative correlation between end-of-period inventories and future price changes.

Calibrating the model to the parameters above, implies a value of $\pi_N$ of about 0.75. On the positive side, the resulting correlation between price changes in adjacent periods is fairly large, approximately 0.3. The information friction slows price adjustment. The correlation between inventories and future price changes is now negative, but only marginally so. This correlation is -0.03. The information friction delays price adjustment but not enough to significantly overcome the natural positive correlation between inventories and future price changes.

On the negative side, the smoothing of prices reduces the correlation between volume and the change in price to just under 0.1, and the average size of price changes is now about 0.5% per quarter. Moreover, the persistence of price changes is fairly short-lived. The correlation between $p_t - p_{t-1}$ and $p_{t-2} - p_{t-3}$ is negative. It remains an open question if combining trading frictions with serial correlation in the fundamentals can produce values that match the data.

This version of the model has implications for the relationship between price and time on the market. At any given time, higher prices will be associated with lower probability of sale and hence greater time on the market. Across periods, however, prices will tend to be higher in tighter markets in which time to sale is low.

There are also implications for price dispersion. Price dispersion is positively correlated with volume. When volume is high, sales work their way further up the pricing ladder. Initial sales are at low prices, and final sales are at relatively high prices. We know of no evidence on this implication of the model.

6. CONCLUSION

We have constructed a model of trading frictions and used it to price houses. The model leads to a simple characterization of equilibrium prices. A robust feature of this model is the positive correlation between vacancies and future price increases. This correlation is likely to be a feature of any model with vacancies. Owners of unsold homes must be compensated for waiting.

The correlation between vacancies and future price appreciation makes it difficult to generate serial correlation in price changes. Positive shocks tend to reduce vacancies
generating expectations of price declines. We have shown that the introduction of 
demand uncertainty and posted prices, both realistic features of real estate markets, 
can overcome some of these difficulties in the short run. 

The model abstracts from several features that may hinder its ability to match 
these facts. The only source of uncertainty is random shocks to supply and demand. 
Adding serial correlation or even nonstationarity to the driving process could improve 
its ability to generate serial correlation in price changes. The model abstracts from 
that these features may generate volatility and a correlation between price and volume. 
It remains an open question whether our mechanism in combination with these or 
other mechanisms can explain can generate realistic housing cycles.

APPENDIX

A.1 Sellers’ Optimal Policy

Let $W$ be an $(N + 1) \times 1$ vector that gives the value of the optimal selling strategy 
when there are $e$ unsold homes at the end of the period. Note that the linearity of 
utility and the lack of borrowing constraints make the problem of selling a home separable.

The Bellman equation is:

$$W_e = \max \{ p_e, -l + \beta x_e' Pe W \},$$

where $x_e$ is a column vector of zeros with a one in the $(e + 1)$st row. Sellers receive 
the maximum of the current price $p_e$ or the value of waiting $-l + \beta x_e' Pe W$.

There are two cases to consider: $e = 0$ and $e > 0$. If $e = 0$, and every seller makes 
a sale. The current price exceeds the value of waiting. Sellers in this case behave 
collectively as monopolists and set price at the maximum that buyers’ are willing 
to pay:

$$W_0 = p_0 = \beta v - V_0,$$

where $V_0$ is the value of owning a home when $e = 0$, and $p_0$ is the price in this case.

If $e > 0$, there are more sellers than buyers. Some homes go unsold. With Bertrand 
competition among identical sellers, sellers must be indifferent between the two 
options. It follows that $W_e = p_e = -l + \beta Pe W$ and $p_e$. Eliminating $W_e$, yields:

$$p_e = -l + \beta x_e' Pe p.$$
A.2 Derivation of Equation (7)

Consider a market with \( b \) sellers. Consider the seller with the \( j \)th highest price. This seller makes a sale if and only if \( n \geq b + 1 - j \). The seller’s expected return is therefore:

\[
\sum_{n=0}^{b-j} \phi(n; N) \beta J(b - n) + \sum_{n=b+1-j}^{\infty} \phi(n; N) p_{j-1}.
\]

The first term gives the return in states in which no sale is made. In this case, the seller must wait to make a sale in the next period and receives a continuation value that we denote by \( J(e) \) where \( e \) is the end-of-period supply of unsold homes in the market. The second term gives the value of a sale. Recall that we are indexing price by \( j-1 \). The highest price is \( p_0 \).

The seller must be indifferent between this return and trading places with the seller with the next highest price. This seller’s expected return is

\[
\sum_{n=0}^{b+1-j} \phi(n; N) \beta J(b - n) + \sum_{n=b+2-j}^{\infty} \phi(n; N) p_j.
\]

Equating these:

\[
\sum_{n=b+1-j}^{\infty} \phi(n; N) p_{j-1} = \phi(b + 1 - j; N) \beta J(j - 1) + \sum_{n=b+2-j}^{\infty} \phi(n; N) p_j.
\]

Both \( b \) and \( j \) appear in this expression. Exponential search reduces the state variables by one. Setting the two returns equal and imposing exponential search, we arrive at

\[
p_{j-1} = \pi p_j + (1 - \pi) \beta J(j - 1).
\]

In the case of the second highest price

\[
p_1 = \pi p_0 + (1 - \pi) \beta J(1).
\]

With exponential search, one buyer is guaranteed to arrive each period. Since sellers are indifferent between all prices, the value to waiting is equal to the value of making a sale in the next period at the lowest price. Hence, \( J(j - 1) = x'_{j-1} P_{e} p \).

A.3 Proof of the Proposition

Proof. Let \( \hat{p}^0 = [V_0 - v, 0, \ldots, 0]' \), and let \( \hat{p}^i = -l + \beta P_{e} \hat{p}^{i-1} \). Standard dynamic programming arguments imply that \( \hat{p}^i \to \hat{p} \).

Now let \( p^0 = P_n \hat{p}^0 \) and suppose that for all \( j \leq i - 1 \), \( p^j = P_n \hat{p}^j \). We show that \( p^i = P_n \hat{p}^i \). The first row of \( P_n \hat{p}^i \) is \( V_0 - v \). This is also the first row of \( p^i \). Suppose
that the $k - 1$st rows of $P_n \hat{p}^i$ and $p^i$ are equal. The $k$th row of $P_n \hat{p}^i$ is

$$
(P_n \hat{p}^i)(k) = \pi (P_n \hat{p}^i)(k - 1) + (1 - \pi)(-l + \beta x_k P_n \hat{p}^i(k - 1))
$$

$$
= \pi \hat{p}^i(k) + (1 - \pi)(-l + \beta x_k P_n \hat{p}^i(k - 1))
$$

$$
= \pi \hat{p}^i(k) + (1 - \pi)(-l + \beta x_k P_n p^{i-1})
$$

$$
= p(k)
$$

The equality of $P_n \hat{p}^i$ and $p^i$ follows by induction. The convergence of $P_n \hat{p}^i$, establishes the proposition. □

LITERATURE CITED


